Adjoint method for lead-fields computation in MEEG.

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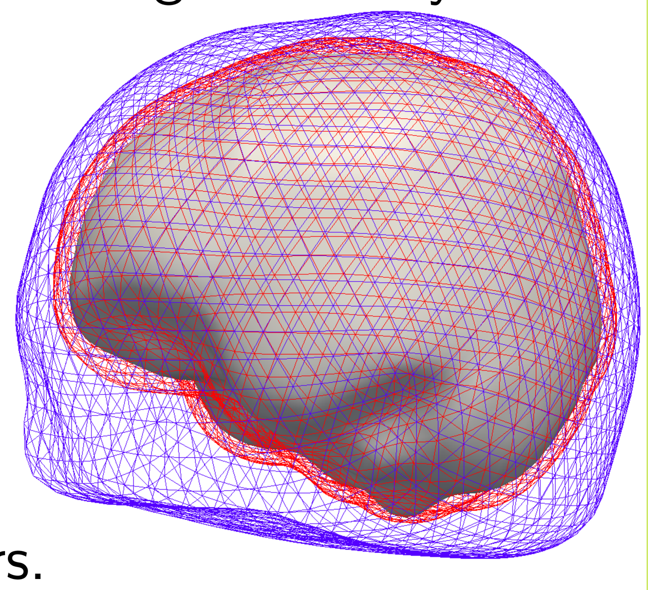
Functional brain imaging with M/EEG (Magneto/Electro-EncephaloGraphy) requires first to compute the **forward problem whose solution is called lead-field matrix**. A line of this lead-field matrix corresponds to a sensor: it expresses the linear relation between each source and this sensor. The number of sources (thousands) largely exceeds the number of sensors (up to 256 electrodes for EEG, and less than 600 squids for MEG). When solving the forward problem with a BEM (Boundary Element Method), the **lead-field matrix is generally computed** column-by-column, i.e. *source by source*, which **represents $\mathbf{n_{sources}}$ resolutions** of the forward problem. Using the adjoint operator of the forward problem [3], one can **reduce the computations to $\mathbf{n_{sensors}}$ resolutions**, as in [4,5,6]. OpenMEEG now implements this functionality, and this poster shows its benefits in terms of memory and computation time.

The forward EEG problem simulates the electric potential V generated by a primary current source distribution J_p.
When the conductivity σ is piecewise constant, the BEM can be used to solve the following equations:

Poisson eq.: $\begin{cases} \nabla\cdot(\sigma\,\nabla V) = \nabla\cdot J_p & \text{in the head volume} \\ (\sigma\,\nabla V)\cdot n = 0 & \text{on the scalp surface} \end{cases}$

Biot-Savart eq.: $\mathbf{B}(\mathbf{r}) = \dfrac{\mu_0}{4\pi}\displaystyle\int (J_p(\mathbf{r}') - \sigma\,\nabla V(\mathbf{r}')) \times \frac{\mathbf{r}-\mathbf{r}'}{\|\mathbf{r}-\mathbf{r}'\|^3} d\mathbf{r}'$

The open-source software OpenMEEG implements the symmetric BEM [1] , and has been shown in [2] to achieve high accuracy.
The adjoint approach is available as of release 2.1 from `http://openmeeg.gforge.inria.fr`.
Matrices used in OpenMEEG are:



- $\mathbf{H}$: head matrix modeling the electrical volume conduction in a specific subject head.
- $\mathbf{X}$: the symmetric BEM's variables (the potential and the normal current discretized on each surface).
- $\mathbf{Y}_{EEG}$, $\mathbf{Z}_{MEG}$: the adjoint EEG and MEG intermediate variables.
- $\mathbf{D}$: right hand side of the Poisson equation, i.e, the projection of the sources onto the BEM surfaces.
- $\mathbf{S}_{EEG}$: selection matrix which computes, given the system unknowns, the potential at EEG sensors.
- $\mathbf{S}_{MEG}$: operator which, given the system unknowns, computes the passive current contribution at MEG sensors.
- $\mathbf{T}_{MEG}$: primary source term in the Biot-Savart equation.
- $\mathbf{L}_{EEG}, \mathbf{L}_{MEG}$: the desired EEG and MEG lead-field matrices.

Conventional approach

Solve the system: $\mathbf{H}\cdot\mathbf{X} = \mathbf{D}$
$\iff$ system of $N \times n_{sources}$ unknowns
Then apply the selection to get the leadfields:

$$\mathbf{L}_{EEG} = \mathbf{S}_{EEG}\cdot\mathbf{X} \qquad \mathbf{L}_{MEG} = \mathbf{S}_{MEG}\cdot\mathbf{X} + \mathbf{T}_{MEG}$$

Adjoint approach

Solve the system: $\mathbf{H}^T\cdot(\mathbf{Y}_{EEG} \quad \mathbf{Z}_{EEG}) = (\mathbf{S}^T_{EEG} \quad \mathbf{S}^T_{MEG})$
$\iff$ system of $N \times n_{sensors}$ unknowns
Then compute the leadfields line-by-line:

$$\mathbf{L}^T_{EEG} = \mathbf{D}^T\cdot\mathbf{Y}_{EEG} \qquad \mathbf{L}^T_{MEG} = \mathbf{D}^T\cdot\mathbf{Z}_{EEG} + \mathbf{T}^T_{MEG}$$

Numerical experiment: EEG and MEG lead-fields were computed for a 3-layer model with $2,562$ vertices per mesh (displayed above), and $20,484$ sources (SPM canonical mesh) using the conventional and the adjoint approaches. Computations were done on a 8-processor computer. Note that the system resolution for the conventional approach was done inverting $\mathbf{H}$ (N^2 variables) which with current Lapack is not multi-threaded, and nor is the resolution of the adjoint system (also using Lapack solver).

fig.1: Computation time in seconds

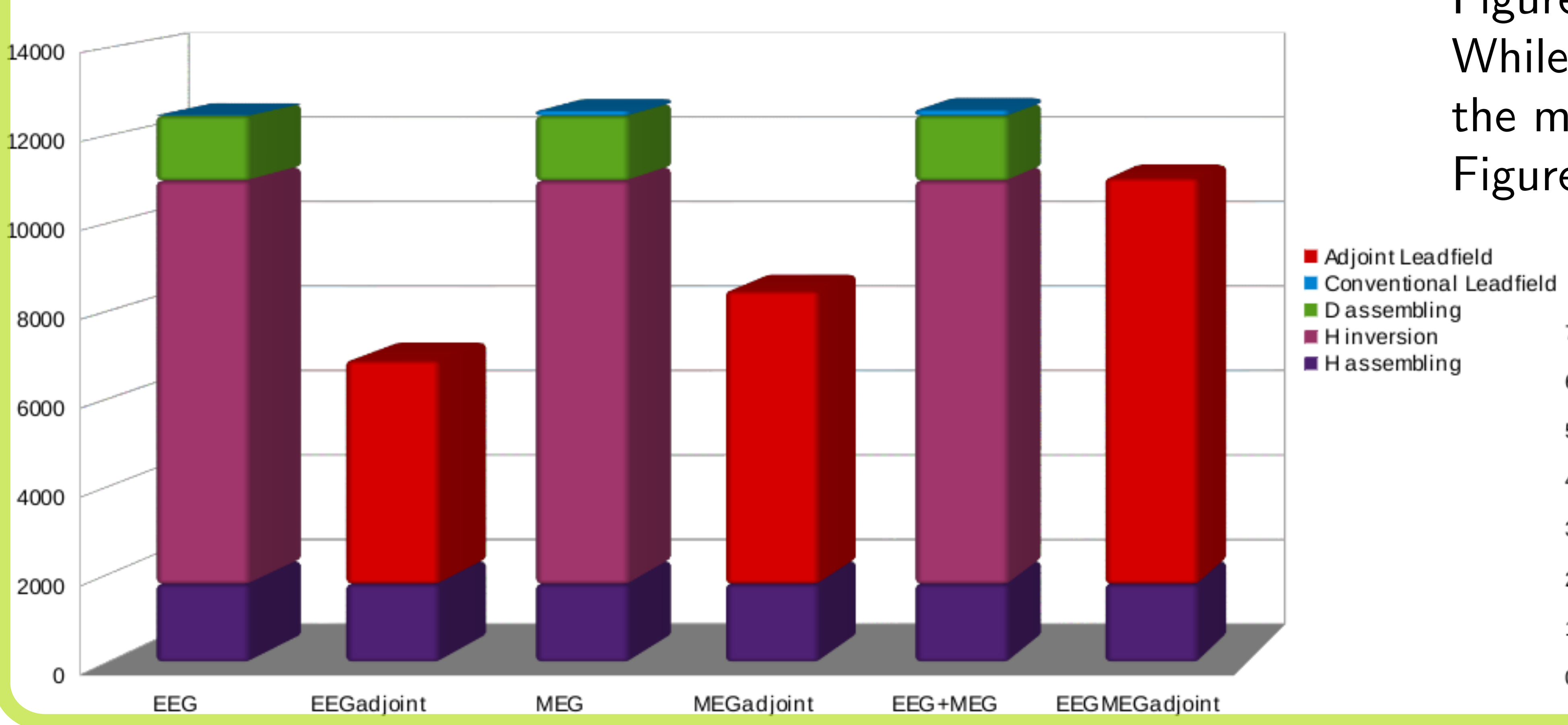
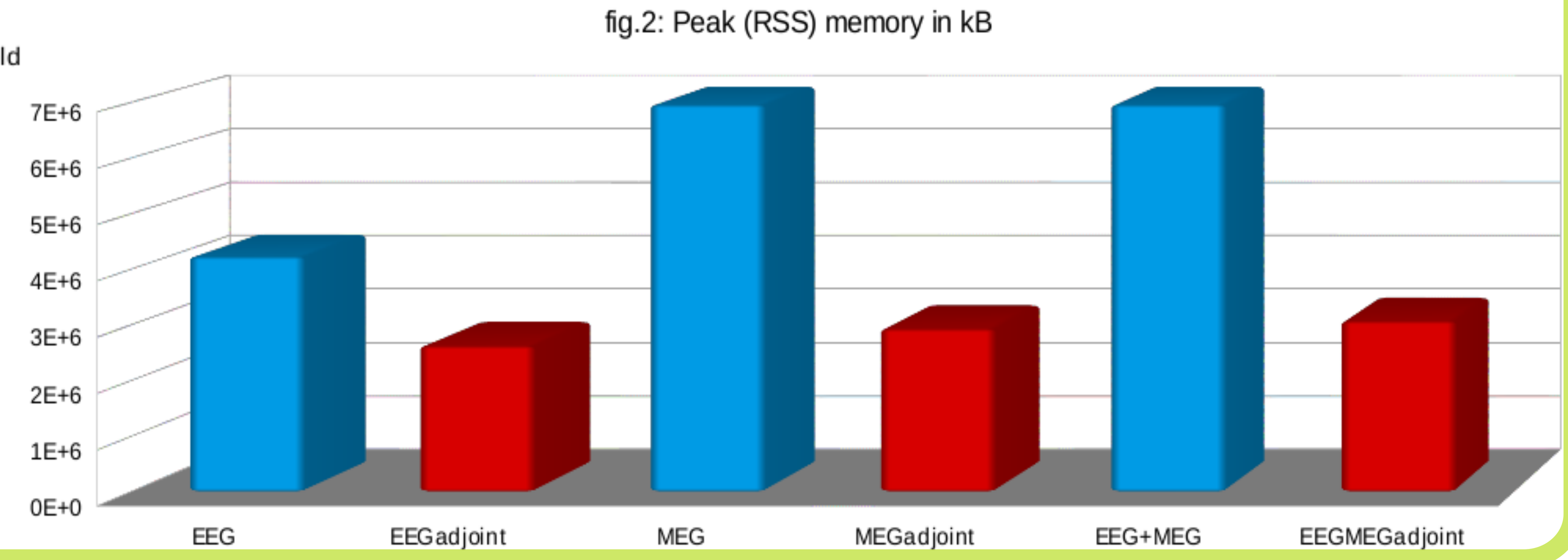


Figure 1. shows the time repartition when computing a lead-field. While the lead-field assembly time (in light blue) is negligible, it is the most memory consuming part.
Figure 2. shows the peak total memory RSS (Resident Set Size).

fig.2: Peak (RSS) memory in kB



Compared to the conventional approach, the adjoint operator associated to the Poisson equation allows to spare computations and produces identical results. As the sensor space is always smaller than the source space, it is generally faster to use the adjoint approach, except in the case of a subject going through several sessions with different sensor locations. Note also, that bigger computational problems can be handled since the size of the system to be solved increases in N, and not N^2 as in the conventional approach.

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